(*) Definitions

1. The function $F(x)$ is an ***antiderivative*** of the function $f(x)$ if $F'(x) = f(x)$.

2. The collection of all antiderivatives of a function $f(x)$ is called the ***indefinite integral*** of $f(x)$,[†] and denoted by

$$\int f(x)\,dx.$$

(*) Comments/Facts

(i) Every continuous function has antiderivatives.

(ii) If a function $f(x)$ has the antiderivative $F(x)$, then for any constant C, the function $F(x)+C$ is also an antiderivative of $f(x)$. Quasi-conversely, if $F(x)$ and $G(x)$ are both antiderivatives of $f(x)$, then $G(x) = F(x) + C$ for some constant C. In other words, if $f(x)$ has an antiderivative, then it has infinitely many and they all have the form $F(x)+C$, where $F(x)$ is any one antiderivative of $f(x)$.

(iii) In terms of the notation for the indefinite integral of $f(x)$, we write

$$\int f(x)\,dx = F(x) + C,$$

where $F(x)$ is any one antiderivative of $f(x)$ and C (the *constant of integration*) is an unspecified constant than can take any value.

(iv) The reason that a factor of dx is included in the indefinite integral is due to the (deep and important) relation between antiderivatives and ***definite integrals***, about which we will learn in a week or so.

(*) Integration Rules/Formulas (the basic ones)

(a) $\displaystyle\int f(x) \pm g(x)\,dx = \left(\int f(x)\,dx\right) \pm \left(\int g(x)\,dx\right),$

i.e., *the integral of a sum is equal to the sum of the integrals.*

(b) $\displaystyle\int af(x)\,dx = a\int f(x)\,dx,$

i.e., *the integral of a constant multiple is the constant multiple of the integral, where* $a \neq 0$.

(c) $\displaystyle\int x^k\,dx = \frac{x^{k+1}}{k+1} + C$, *for all* $k \neq -1$. *The case* $k = -1$ *is covered next.*

(d) $\displaystyle\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C.$

(e) $\displaystyle\int e^x\,dx = e^x + C.$

[†]Technically it is the indefinite integral of the differential $f(x)\,dx$.