Final Exam

Instructions

- There are 6 questions worth a total of 54 points. 100%=50 points.
- No notes or books. A table of integration formulas is provided
- You ***may*** use a simple scientific calculator. ***No*** graphing or programmable calculators.
- ***Take your time. Answer each question completely. Check your answers.***
- ***For full credit—explain/show your work.***

Good Luck!!!

NAME: ____

Problem	Score
1	/9
2	/9
3	/9
4	/9
5	/9
6	/9
Total	/54

Selected Integration Formulas

Basic rules.

1. $\displaystyle\int u^k\,du = \frac{u^{k+1}}{k+1} + C, \quad k \neq -1.$

2. $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C.$

3. $\displaystyle\int e^u\,du = e^u + C.$

4. $\displaystyle\int f(u) \pm g(u)\,du = \int f(u)\,du \pm \int g(u)\,du.$

5. $\displaystyle\int c\cdot f(u)\,du = c\cdot\int f(u)\,du.$

Rational forms containing $(\mathbf{a}+\mathbf{bu})$.

6. $\displaystyle\int \frac{du}{a+bu} = \frac{1}{b}\ln|a+bu| + C.$

7. $\displaystyle\int \frac{u\,du}{a+bu} = \frac{u}{b} - \frac{a}{b^2}\ln|a+bu| + C.$

8. $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3}\ln|a+bu| + C.$

9. $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a+bu)} - \frac{2a}{b^3}\ln|a+bu| + C.$

Forms containing $\sqrt{\mathbf{a}+\mathbf{bu}}$.

10. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2(3bu-2a)(a+bu)^{3/2}}{15b^2} + C.$

11. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2(bu-2a)\sqrt{a+bu}}{3b^2} + C.$

12. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2(3b^2u^2-4abu+8a^2)\sqrt{a+bu}}{15b^3} + C.$

Exponential and logarithmic forms.

13. $\displaystyle\int e^{au}\,du = \frac{e^{au}}{a} + C.$

14. $\displaystyle\int ue^{au}\,du = \frac{e^{au}}{a^2}(au-1) + C.$

15. $\displaystyle\int u^n e^{au}\,du = \frac{u^n e^{au}}{a} - \frac{n}{a}\int u^{n-1}e^{au}\,du\,.$

16. $\displaystyle\int u^n \ln u\,du = \frac{u^{n+1}\ln u}{n+1} - \frac{u^{n+1}}{(n+1)^2} + C, \qquad n \neq -1.$

1. (9 pts) A nation's marginal propensity to consume is given by

$$\frac{dC}{dY} = \frac{8Y + 10}{9Y + 2},$$

where annual consumption (C) and annual income (Y) are both measured in billion's of dollars. Find the changes in national consumption and national savings when income increases from $Y = 10$ to $Y = 20$.

Suggestion: Use the table to save time.

2. (9 pts) The supply and demand functions for a given market are

$$\text{Supply: } p = 10 + \frac{q}{2} \ \text{ and } \ \text{Demand: } p = 40 - \frac{q^2}{60}.$$

Find the Consumers' and Producers' surplus at equilibrium for this market.

3. A household's utility function is given by

$$U(x, y, z) = 7 \ln x + 8 \ln y + 10 \ln z,$$

where x, y and z are the quantities of products X, Y and Z, respectively, consumed by the household each month. The prices per unit for these three goods are $p_x = \$5$, $p_y = \$16$ and $p_z = \$20$, respectively.

(a) (6 pts) Find the quantities of X, Y and Z that should be consumed each month to maximize the household's utility, given that their monthly disposable income is $Y_d = \$4000$.

(b) (3 pts) By approximately how much will the household's maximum utility change if p_x increases to \$5.50, assuming that all else stays the same?

Suggestion: The envelope theorem and linear approximation may be useful here.

4. The production function for ACME WIDGETS is given by

$$Q = 20K^{0.7}L^{0.4},$$

where Q is the number of widgets ACME produces in one year, K is the number of units of capital input and L is the number of units of labor input ACME uses to produce their widgets.

The price per unit of capital input is $p_K = \$5,000$ and the price per unit of labor input is $p_L = \$2000$.

(a) (6 pts) Find the levels capital and labor input that ***minimize the cost*** of producing $10,000$ widgets. What is the corresponding minimum cost?

(b) (3 pts) By approximately how much will the firm's minimum cost increase, if they increase their output by $\Delta Q = 100$ widgets? Show your work.

5. The monthly demand q for a firm's product in a certain market is related to the price of their product, p, and the average monthly disposable income in the market, y_d, by the equation:

$$q = 50 \ln \left(6y_d - 2p^3\right).$$

(a) (6 pts) Compute q, $\partial q/\partial p$, and $\partial q/\partial y_d$ when $p = 10$ and $y_d = 3500$.

(b) (3 pts) Use linear approximation to estimate the change in demand for the firm's product if average monthly disposable income increases to $y_d = 3800$ and the firm increases the price to $p = 11$.

6. (9 pts) A monopolistic firm produces two competing varieties of energy drink, *high-octane* (type A) and *regular* (type B). It costs the firm \$5 per gallon to produce the *high-octane* drink and \$3 per gallon to produce the *regular* one. The quantities, Q_A and Q_B (both measured in gallons), of each variety that can be sold per day are given by the joint demand functions

$$\begin{aligned} Q_A &= 300(P_B - 2P_A + 11) \\ Q_B &= 300(P_A - P_B), \end{aligned}$$

where P_A and P_B are the selling prices per gallon of *high-octane* and *regular*, respectively.

Find the selling prices that will maximize the firm's daily profit and the firm's maximum daily profit. Use the second derivative test to justify your claim that the prices you found yield the maximum profit.