

Section 14.6, problem 8:

$$\begin{aligned}
S_n &= \frac{2}{n} \left[\left(\frac{2}{n} \right)^2 + \left(2 \cdot \frac{2}{n} \right)^2 + \left(3 \cdot \frac{2}{n} \right)^2 + \cdots + \left(n \cdot \frac{2}{n} \right)^2 \right] \\
&= \frac{2}{n} \sum_{j=1}^n \left(j \cdot \frac{2}{n} \right)^2 = \frac{2}{n} \sum_{j=1}^n j^2 \cdot \frac{4}{n^2} = \frac{8}{n^3} \sum_{j=1}^n j^2 \\
&= \frac{8}{n^3} \cdot \frac{2n^3 + 3n^2 + n}{6} \quad (\text{using the formula from SN1, or the book}) \\
&= \frac{16n^3}{6n^3} + \frac{12n^2}{6n^3} + \frac{4n}{6n^3} = \frac{8}{3} + \frac{2}{n} + \frac{2}{3n^2}
\end{aligned}$$

and therefore

$$\lim_{n \rightarrow \infty} S_n = \lim_{n \rightarrow \infty} \left(\frac{8}{3} + \frac{2}{n} + \frac{2}{3n^2} \right) = \frac{4}{3} + 0 + 0 = \frac{8}{3}.$$

Section 14.7, problem 12:

$$\int_2^3 \frac{3}{x^2} dx = 3 \int_2^3 x^{-2} dx = 3 \left(\frac{x^{-1}}{-1} \Big|_2^3 \right) = 3 ((-3^{-1}) - (-2^{-1})) = 3 \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{1}{2}.$$

Section 14.7, problem 52:

First the mean, μ :

$$\begin{aligned}
\mu &= \int_0^1 x \cdot f(x) dx = \int_0^1 x \cdot 6(x - x^2) dx \\
&= 6 \int_0^1 x^2 - x^3 dx = 6 \left(\frac{x^3}{3} - \frac{x^4}{4} \Big|_0^1 \right) \\
&= 6 \left(\left(\frac{1}{3} - \frac{1}{4} \right) - (0 - 0) \right) = \frac{1}{2}.
\end{aligned}$$

And now the variance, σ^2 :

$$\begin{aligned}
\sigma^2 &= \int_0^1 (x - \mu)^2 \cdot f(x) dx = \int_0^1 (x - 0.5)^2 \cdot 6(x - x^2) dx \\
&= 6 \int_0^1 (x^2 - x + 0.25)(x - x^2) dx \\
&= 6 \int_0^1 -x^4 + 2x^3 - 1.25x^2 + 0.25x dx \\
&= 6 \left(\frac{-x^5}{5} + \frac{2x^4}{4} - \frac{1.25x^3}{3} + \frac{0.25x^2}{2} \Big|_0^1 \right) \\
&= 6 \left(\left(-\frac{1}{5} + \frac{1}{2} - \frac{5}{12} + \frac{1}{8} \right) - (-0 + 0 - 0 + 0) \right) = \frac{1}{20}.
\end{aligned}$$