AMS/Econ 11B

Homework 4 – Solutions

Section 14.9, problem 60: The Gini coefficient for the Lorenz curve  $f(x) = \frac{11}{12}x^2 + \frac{1}{12}x$  is

$$\gamma = \frac{\int_0^1 x - \left(\frac{11}{12}x^2 + \frac{1}{12}x\right) \, dx}{\frac{1}{2}}$$

Which as we saw in class (you can also look at the notes from the lecture on the course website), simplifies to

$$\gamma = 1 - 2\int_0^1 \frac{11}{12}x^2 + \frac{1}{12}x\,dx = 1 - 2\left[\frac{11}{36}x^3 + \frac{1}{24}x^2\Big|_0^1\right] = 1 - 2\left(\frac{11}{36} + \frac{1}{24}\right) = \frac{11}{36} \approx 0.306$$

## Section 14.10, problem 4:

(1) Point of equilibrium:

$$900-q^2 = 10q+300 \implies q^2+10q-600 = 0 \implies (q-20)(q+30) = 0 \implies q = 20 \text{ or } q = 30$$
  
Equilibrium:  $q^* = 20$  and  $p^* = 900 - 20^2 = 500$ .

(2) Consumers' surplus:

$$CS = \int_0^{q^*} \text{demand} - p^* \, dq = \int_0^{20} 900 - q^2 - 500 \, dq = 400q - \frac{q^3}{3} \Big|_0^{20} = \frac{16000}{3} \approx 5333.33.$$

(3) Producers' surplus:

$$PS = \int_0^{q^*} p^* - \text{supply} \, dq = \int_0^{20} 500 - (10q + 300) \, dq = \left. 200q - 5q^2 \right|_0^{20} = 2000.$$

Section 15.4, problem 8: The average value of  $f(x) = 5/x^2$  on the interval [1,3] is

Avg(f) = 
$$\frac{1}{3-1} \int_{1}^{3} 5x^{-2} dx = \frac{5}{2} \left[ \frac{x^{-1}}{-1} \Big|_{1}^{3} \right] = \frac{5}{2} \left( -\frac{1}{3} - (-1) \right) = \frac{5}{3}.$$

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