

Section 15.3, problem 12: Use formula #14 in Appendix B, with $a = b = 1$:

$$\int x^2 \sqrt{1+x} dx = \frac{2(8 - 12x + 15x^2)(1+x)^{3/2}}{105} + C \quad \left(= \frac{(16 - 24x + 30x^2)(1+x)^{3/2}}{105} + C \right)$$

Section 15.3, problem 58: Use formula #6,¹ with $a = 1$ and $b = -1$:

$$\begin{aligned} n &= -\frac{1}{0.4} \int_{0.3}^{0.1} \frac{dq}{q^2(1-q)} = -\frac{1}{0.4} \left[-\frac{1}{q} - \ln \left| \frac{1-q}{q} \right| \right]_{0.3}^{0.1} \\ &= \frac{1}{0.4} \left[\frac{1}{q} + \ln \left| \frac{1-q}{q} \right| \right]_{0.3}^{0.1} \\ &= \frac{5}{2} \left[\underbrace{(10 + \ln 9)}_{\text{at } q=0.1} - \underbrace{(10/3 + \ln(7/3))}_{\text{at } q=0.3} \right] \\ &\approx 20 \end{aligned}$$

Section 15.5, problem 6:

Separate: $y' = e^x y^3 \implies \frac{dy}{dx} = e^x y^3 \implies y^{-3} dy = e^x dx.$

Integrate: $\int y^{-3} dy = \int e^x dx \implies \frac{y^{-2}}{-2} = e^x + C$

Solve for y: $\frac{y^{-2}}{-2} = e^x + C \implies \frac{1}{y^2} = -2e^x + C \implies y^2 = \frac{1}{C - 2e^x}$

$$\implies y = (C - 2e^x)^{-1/2} \quad \left(= \frac{1}{\pm\sqrt{C - 2e^x}} = \pm\sqrt{\frac{1}{C - 2e^x}} \right)$$

Section 15.5, problem 22:

Separate: $\frac{dr}{dq} = (50 - 4q)e^{-r/5} \implies e^{r/5} dr = (50 - 4q) dq$

Integrate: $\int e^{r/5} dr = \int (50 - 4q) dq \implies \frac{1}{5}e^{r/5} = 50q - 2q^2 + C$

Solve for r: $\frac{1}{5}e^{r/5} = 50q - 2q^2 + C \implies e^{r/5} = 250q - 10q^2 + C \implies \frac{r}{5} = \ln(250q - 10q^2 + C)$
 $\implies r = 5 \ln(250q - 10q^2 + C)$

¹Sorry about the hint in the syllabus — it's correct, but it is more direct to use formula #6, which I didn't notice at the time.

Solve for C: $r(0) = 0 \implies 0 = 5 \ln(C) \implies \ln C = 0$

$$\implies C = 1 \implies r = 5 \ln(250q - 10q^2 + 1)$$

Demand: $p = \frac{r}{q} = \frac{5 \ln(250q - 10q^2 + 1)}{q}$.