AMS/Econ 11B

Homework 6 – Solutions

The assumption of logistic growth means that

Section 15.6, problem 4:

$$N(t) = \frac{M}{1 + be^{-rt}},$$

where in this case is M = 50,000, because the number of students with the ringtone cannot exceed the number of students at the university. I.e.,

$$N(t) = \frac{50000}{1 + be^{-rt}}.$$

When the investigation starts, 500 students have the ringtone, so N(0) = 500, which means that

$$500 = N(0) = \frac{50000}{1 + be^0} \implies 1 + b = \frac{50000}{500} \implies b = 99$$

since $e^0 = 1$. One week later, the number of students with the ringtone is 1500, and assuming that we measure time in weeks, this means that

$$1500 = N(1) = \frac{50000}{1 + 99e^{-r}} \implies 1 + 99e^{-r} = \frac{50000}{1500} \implies e^{-r} = \frac{100/3 - 1}{99} \approx 0.3266$$

It follows that $r = -\ln(0.3266) \approx 1.119$, so the formula that the newspaper publishes is

$$N(t) = \frac{50000}{1 + 99e^{-1.119t}}.$$

Section 17.1, problem 8: $g(w, z) = \sqrt[3]{w^2 + z^2} = (w^2 + z^2)^{1/3}$, so

$$\frac{\partial g}{\partial w} = \frac{1}{3}(w^2 + z^2)^{-2/3} \cdot 2w \qquad \left(= \frac{2w}{3(w^2 + z^2)^{2/3}} \right)$$

and

$$\frac{\partial g}{\partial z} = \frac{1}{3}(w^2 + z^2)^{-2/3} \cdot 2z \qquad \left(= \frac{2z}{3(w^2 + z^2)^{2/3}} \right)$$

Section 17.1, problem 14: $h(x,y) = \frac{\sqrt{x+9}}{x^2y + y^2x} = \frac{(x+9)^{1/2}}{x^2y + xy^2}$, so

$$h_x = \frac{\frac{1}{2}(x+9)^{-1/2}(x^2y+xy^2) - (x+9)^{1/2}(2xy+y^2)}{(x^2y+xy^2)^2} \quad \left(= -\frac{x^2y+xy^2 - 36xy - 18y^2}{2(x+y)^{1/2}(x^2y+xy^2)^2} \right)$$

and

$$h_y = \frac{\partial}{\partial y} \left(\sqrt{x+9} \cdot (x^2y + xy^2)^{-1} \right) = \sqrt{x+9} \cdot \left((-1)(x^2y + xy^2)^{-2}(x^2 + 2xy) \right) = -\frac{\sqrt{x+9}(x^2 + 2xy)}{(x^2y + xy^2)^2}$$

Section 17.2, problem 24: $q_A = 60 - 3p_A - 2p_B$, so $q_A \Big|_{\substack{p_A=5\\p_B=3}} = 39$,

$$\eta_{p_A}\Big|_{p_A=5\atop p_B=3} = \left.\frac{\partial q_A}{\partial p_A} \cdot \frac{p_A}{q_A}\right|_{p_A=5\atop p_B=3} = (-3) \cdot \frac{5}{39} = -\frac{5}{13}$$

and

$$\eta_{p_B}\Big|_{p_A=5\atop p_B=3} = \frac{\partial q_A}{\partial p_B} \cdot \frac{p_A}{q_A}\Big|_{p_A=5\atop p_B=3} = (-2) \cdot \frac{3}{39} = -\frac{2}{13}.$$