

**Section 17.4, problem 6:**  $f(x, y) = \ln(x^2 + y^2) + 2$ .

$$f_x(x, y) = \frac{2x}{x^2 + y^2}$$

$$f_{xx}(x, y) = \frac{2(x^2 + y^2) - 2x \cdot 2x}{(x^2 + y^2)^2} = \frac{2y^2 - 2x^2}{(x^2 + y^2)^2}$$

$$f_{xy}(x, y) = \frac{-2y \cdot 2x}{(x^2 + y^2)^2} = -\frac{4xy}{(x^2 + y^2)^2}$$

**Section 17.4, problem 12.**  $f(x, y, z) = z^2(3x^2 - 4xy^3) = 3x^2z^2 - 4xy^3z^2$ .

$$f_x(x, y, z) = 6xz^2 - 4y^3z^2 \implies f_{xy}(x, y, z) = -12y^2z^2 \implies f_{xyz} = -24y^2z,$$

so

$$f_{yz}(1, 2, 3) = -24 \cdot 4 \cdot 3 = -288.$$

**SN2, problem 4:** Find the quadratic Taylor polynomial for  $H(x, y) = \sqrt{2x + 5y}$ , centered at  $(x_0, y_0) = (3, 2)$ , and use it to approximate  $H(3.25, 2.1) = \sqrt{17}$

- $H(3, 2) = 4$ ;
- $H_x = \frac{1}{2}(2x + 5y)^{-1/2} \cdot 2 = (2x + 5y)^{-1/2}$ , so  $H_x(3, 2) = \frac{1}{4}$ ;
- $H_y = \frac{1}{2}(2x + 5y)^{-1/2} \cdot 5 = \frac{5}{2}(2x + 5y)^{-1/2}$ , so  $H_y(3, 2) = \frac{5}{8}$ ;
- $H_{xx} = -\frac{1}{2}(2x + 5y)^{-3/2} \cdot 2 = -(2x + 5y)^{-3/2}$ , so  $H_{xx}(3, 2) = -\frac{1}{64}$ ;
- $H_{xy} = -\frac{1}{2}(2x + 5y)^{-3/2} \cdot 5 = -\frac{5}{2}(2x + 5y)^{-3/2}$ , so  $H_{xy}(3, 2) = -\frac{5}{128}$ ;
- $H_{yy} = -\frac{1}{2} \cdot \frac{5}{2}(2x + 5y)^{-1/2} \cdot 5 = -\frac{25}{4}(2x + 5y)^{-3/2}$  so  $H_{yy}(3, 2) = -\frac{25}{256}$ .

This means that the quadratic Taylor polynomial for  $H(x, y)$  centered at  $(3, 2)$  is

$$T(x, y) = 4 + \frac{1}{4}(x - 3) + \frac{5}{8}(y - 2) - \frac{1}{128}(x - 3)^2 - \frac{5}{128}(x - 3)(y - 2) - \frac{25}{512}(y - 2)^2,$$

and

$$\sqrt{17} = H(3.25, 2.1) \approx T(3.25, 2.1) = 4 + \frac{1}{16} + \frac{1}{16} - \frac{1}{2048} - \frac{1}{1024} - \frac{1}{2048} = \frac{2111}{512} \quad (\approx 4.123).$$