Section 17.6, problem 10: $f(x, y)=2 x^{2}+\frac{3}{2} y^{2}+3 x y-10 x-9 y+2$
(1) Finding critical point(s):

$$
\begin{array}{ll}
f_{x}=0 \Longrightarrow 4 x+3 y-10=0 & \Longrightarrow 4 x+3 y=10 \\
f_{y}=0 \Longrightarrow 3 y+3 x-9=0 & \Longrightarrow 3 x+3 y=9
\end{array}
$$

Subtracting the second equation on the right from the first, gives $x=1$, and plugging this into the first equation give $4+3 y=10 \Longrightarrow y=2$. So there is one critical point: $\left(x^{*}, y^{*}\right)=(1,2)$.
(2) Second derivative test: $f_{x x}=4, f_{x y}=3$ and $f_{y y}=3$, so the discriminant in this case is

$$
D(1,2)=f_{x x}(1,2) f_{y y}(1,2)-f_{x y}^{2}(1,2)=12-9=3>0
$$

and since $f_{x x}>0$, the point $(1,2)$ yields the minimum value $f(1,2)=-12$.

## Section 17.6, problem 26:

(1) The profit function:

$$
\begin{aligned}
\Pi & =\overbrace{p_{A} q_{A}+p_{B} q_{B}}^{\text {revenue }}-(\overbrace{2 q_{A}+4 q_{B}}^{\text {cost }}) \\
& =p_{A}\left(16-p_{A}+p_{B}\right)+p_{B}\left(24+2 p_{A}-4 p_{B}\right)-2\left(16-p_{A}+p_{B}\right)-4\left(24+2 q_{A}-4 q_{B}\right) \\
& =-p_{A}^{2}+3 p_{A} p_{B}-4 p_{B}^{2}+10 p_{A}+38 p_{B}-128
\end{aligned}
$$

(2) Critical point:

$$
\begin{aligned}
& \Pi_{p_{A}}=0 \Longrightarrow-2 p_{A}+3 p_{B}+10=0 \\
& \Pi_{p_{B}}=0 \Longrightarrow 3 p_{A}-8 p_{B}+38=0
\end{aligned}
$$

Adding 3 times the first equation to 2 times the second one gives

$$
-7 p_{B}+106=0 \Longrightarrow p_{B}^{*}=\frac{106}{7} \quad(\approx 15.14)
$$

and substituting $p_{B}^{*}$ into the first equation gives

$$
-2 p_{A}+\frac{318}{7}+10=0 \Longrightarrow p_{A}^{*}=\frac{194}{7} \quad(\approx 27.71)
$$

I.e., the critical point is $\left(p_{A}^{*}, p_{B}^{*}\right)=\left(\frac{194}{7}, \frac{106}{7}\right)$. The corresponding critical output levels are

$$
q_{A}^{*}=16-p_{A}^{*}+p_{B}^{*}=\frac{24}{7} \quad(\approx 3.43) \quad \text { and } \quad q_{B}^{*}=24+2 p_{A}-4 p_{B}^{*}=\frac{132}{7} \quad(\approx 18.86)
$$

(3) Verifying a maximum, using the second derivative test:

$$
\Pi_{p_{A} p_{A}}=-2, \quad \Pi_{p_{A} p_{B}}=3 \quad \text { and } \quad \Pi_{p_{B} p_{B}}=-8
$$

so

$$
D=(-2)(-8)-3^{2}=7>0
$$

and since $\Pi_{p_{A} p_{A}}=-2<0$, the critical output levels above do maximize the profit.

