Section 17.7, problem 16: The Lagrangian is

$$F(l,k,\lambda) = 20l + 25k - l^2 - 3k^2 - \lambda(2l + 4k - 50).$$

The first order conditions:

$$F_l = 0 \implies 20 - 2l - 2\lambda = 0 \implies \lambda = 10 - l$$

and

$$F_k = 0 \implies 25 - 6k - 4\lambda = 0 \implies \lambda = \frac{25 - 6k}{4}.$$

Next

$$\lambda = \lambda \implies 10 - l = \frac{25 - 6k}{4} \implies 40 - 4l = 25 - 6k \implies k = \frac{4l - 15}{6}.$$

Plugging this into the constraint, we find that

$$2l + 4k = 50 \implies 2l + 4 \cdot \frac{4l - 15}{6} = 50 \implies 28l = 360 \implies l^* = \frac{360}{28} = \frac{90}{7}$$

and

$$k^* = \frac{4l^* - 15}{6} = \frac{255}{42}.$$

Finally, the firm's maximum output subject to the budget constraint is

$$f^* = 20l^* + 25k^* - (l^*)^2 - 3(k^*)^2 \approx 133.$$

Section 17.7, problem 18: The constraint in this case is

$$25l + 69k = 25875$$

and the Lagrangian is

$$F(l,k,\lambda) = 6l^{2/5}k^{3/5} - \lambda(25l + 69k - 25875).$$

First order conditions:

$$F_l = 0 \implies \frac{12}{5}l^{-3/5}k^{3/5} - 25\lambda = 0 \implies \lambda = \frac{12k^{3/5}}{125l^{3/5}}$$

and

$$F_k = 0 \implies \frac{18}{5}l^{2/5}k^{-2/5} - 69\lambda = 0 \implies \lambda = \frac{18l^{2/5}}{345k^{2/5}}.$$

Next

$$\lambda = \lambda \implies \frac{12k^{3/5}}{125l^{3/5}} = \frac{18l^{2/5}}{345k^{2/5}} \implies 4140k = 2250l \implies k = \frac{25}{46}l.$$

Plugging this into the constraint gives

$$25l + 69k = 25875 \implies 25l + 69 \cdot \frac{25}{46}l = 25875 \implies l^* = 414$$

and

$$k^* = \frac{25}{46} l^* = 225.$$

SN 5, problem 4: At the end of Example 11, the maximum utility U^* is given as

$$U^* = a \ln \left(\frac{Y_d}{p_A} \left(\frac{a}{a+b} \right) \right) + b \ln \left(\frac{Y_d}{p_B} \left(\frac{b}{a+b} \right) \right),$$

and simplifying this, using properties of the logarithm functions shows that

$$U^* = \overbrace{(a+b)}^{\alpha} \ln Y_d + \overbrace{\left[a \ln \left(\frac{a}{a+b} \right) + b \ln \left(\frac{b}{a+b} \right) - a \ln p_A - b \ln p_B \right]}^{\beta} = \alpha \ln Y_d + \beta.$$