(1) Compute the definite integrals below.

(a) \[\int_1^4 \frac{4v}{3v^2-1} dv = \frac{2}{3} \int_2^{47} \frac{1}{u} du = \frac{2}{3} \ln |u| \bigg|_2^{47} = \frac{2}{3} \ln 47 - \frac{2}{3} \ln 2 \approx 2.105\]

I used the substitution \(u = 3v^2 - 1\), \(du = 6v \, dv \implies 4v \, dv = \frac{2}{3} \, du\), and the limits of integration changed as well: \(v = 1 \implies u = 3 \cdot 1^2 - 1 = 2\) and \(v = 4 \implies u = 3 \cdot 4^2 - 1 = 47\).

(b) \[\int_0^5 2t^3 + 4t - 5 \, dt = \frac{1}{2} t^4 + 2t^2 - 5t \bigg|_0^5 = 337 - 0 = 337\]

(2) The marginal propensity to save for a small nation is given by

\[\frac{dS}{dY} = \frac{Y+1}{9Y+10}\]

where the nation’s annual saving \(S\) and annual income \(Y\) are both measured in billions of dollars.

Find the total change in the annual saving and annual consumption if annual income increases from $5 billion to $8 billion.

Step 1. Integrate, using the substitution \(u = 9Y + 10 = \implies Y = \frac{1}{9} (u-10)\) and \(du = 9 \, dY \implies dY = \frac{1}{9} \, du\):

\[\int \frac{Y+1}{9Y+10} \, dY = \frac{1}{9} \int \frac{1}{u} \, du = \frac{1}{9} \int \frac{1}{u} - \frac{1}{9} \cdot u^{-1} \, du = \frac{1}{81} u - \frac{1}{81} \ln |u| + K = \frac{1}{81} (9Y + 10) - \frac{1}{81} \ln |9Y + 10| + K\]

\[\implies S = \frac{1}{9} Y - \frac{1}{81} \ln |9Y + 10| + K\]

Step 2. Calculate \(\Delta S = S(8) - S(5)\):

\[S(8) - S(5) = \left(\frac{1}{9} \cdot 8 - \frac{1}{81} \ln |9 \cdot 8 + 10| + K\right) - \left(\frac{1}{9} \cdot 5 - \frac{1}{81} \ln |9 \cdot 5 + 10| + K\right) \approx 0.3284\]

Comment: According to the FTC, \(\Delta S = \int_5^8 \frac{Y+1}{9Y+10} \, dY\), which is what we just calculated above.

Step 3. Calculate \(\Delta C = \Delta Y - \Delta S \approx 3 - 0.3284 = 2.6716\).

(3) Find the demand equation for the firm whose marginal revenue function is

\[\frac{dr}{dq} = \frac{750}{(3q+2)^3}\]

Step 1. Integrate: \[\int \frac{750}{(3q+2)^3} \, dq = 250 \int u^{-3} \, du = 250 \cdot \frac{u^{-2}}{-2} + C = -125(3q+2)^{-2} + C\]

using the substitution \(u = 3q + 2\), \(du = 3 \, dq \implies 750 \, dq = 250 \, du\). This means that the firm’s revenue function has the form

\[r = -125(3q+2)^{-2} + C.\]
Step 2. Solve for $C$:

\[ 0 = r(0) = -125(0 + 2)^{-2} + C = C - \frac{125}{4} \implies C = \frac{125}{4}. \]
So the firm’s revenue function is

\[ r = \frac{125}{4} - 125(3q + 2)^{-2} \quad \left(= \frac{1125q^2 + 1500q}{4(3q + 2)^2}\right). \]

Step 3. Divide by $q$ to find the demand equation:

\[ p = \frac{r}{q} = \frac{125}{4q} - \frac{125}{q(3q + 2)^2} \quad \left(= \frac{1125q + 1500}{4(3q + 2)^2}\right). \]

(4) Compute the indefinite integrals below.

(a) \[ \int 3e^t(4e^t - 1)^3/7 \, dt = \frac{3}{4} \int u^{3/7} \, du = \frac{3}{4} \cdot \frac{u^{10/7}}{10/7} + C = \frac{21}{40} (4e^t - 1)^{10/7} + C, \]
using the substitution $u = 4e^t - 1$, $du = 4e^t \, dt \implies 3e^t \, dt = \frac{3}{4} \, du.$

(b) \[ \int \frac{5x^3 + 3x^2 + 2}{4x} \, dx = \int \frac{5}{4}x^2 + \frac{3}{4}x + \frac{1}{2}x^{-1} \, dx = \frac{5}{4} \cdot \frac{x^3}{3} + \frac{3}{4} \cdot \frac{x^2}{2} + \frac{1}{2} \ln|x| + C = \frac{5}{12}x^3 + \frac{3}{8}x^2 + \frac{1}{2} \ln|x| + C. \]